## Physics Department

## Waves Questions

## Basic Waves

## Wave Properties

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## WAVES: FREQUENCY, TIME PERIOD, WAVELENGTH, SPEED.

1. Calculate the periods of oscillation for the following frequencies:
a) 2.0 Hz
b) 30 kHz
c) 25 MHz
d) 4.0 GHz
2. Calculate the frequencies that correspond to the following periods. Give your answers to appropriate significant figures in standard form using appropriate prefixes:
a) 1.4 ms
b) 120 s
c) 48 ns
d) $5.0 \mu \mathrm{~s}$
3. A wave travels at $330 \mathrm{~ms}^{-1}$. the wavelength is found to be 2.4 m . Calculate
a) the period of oscillation of a article in the medium transmitting the wave
b) the frequency of oscillation of the source transmitting the wave.
4. Copy and complete the following table

| Frequency | Velocity | Wavelength |
| :--- | :--- | :--- |
| 15 Hz | $120 \mathrm{~ms}^{-1}$. |  |
|  | $5000 \mathrm{~ms}^{-1}$. | 1.3 mm |
| $1.2 \times 10^{15} \mathrm{~Hz}$ | $2.0 \times 10^{8} \mathrm{~ms}^{-1}$. |  |
| 230 Hz |  | 1.48 m |
|  | $3.0 \times 10^{8} \mathrm{~ms}^{-1}$. | 2.8 cm |

5. Define the following terms, when applied to waves:
a) frequency
b) time period c) wavelength
d) wave speed

## Problems on Describing Waves

1. The graph below represents the displacement against position of a water wave at a particular time. Find (i) the amplitude $\mathrm{A}_{\text {; }}$ (ii) the wavelength $\lambda$.

2. The graph below represents the same water wave as in the first graph. Find (i) the amplitude $A_{\text {; }}$ (ii) the time period $T_{\text {; }}$ (iii) the frequency $f$.

3. Using your results from questions 1 and 2 calculate the speed $v$ of the water wave.
4. The wavelength of light emitted by a sodium vapour lamp is $5 \times 10^{-7} \mathrm{~m}$. Given that the speed of light waves in a vacuum is $3 \times 10^{8} \mathrm{~ms}^{-1}$ calculate the frequency of the waves.
5. A wave has the following properties:

Amplitude $\mathrm{A}=15 \mathrm{~mm}$
Frequency $f=2 \mathrm{~Hz}$
Speed v $=10 \mathrm{~cm} \mathrm{~s}^{-1}$
Plot a displacement/position graph and a displacement/time graph for the wave.
6. Radio 4 Long Wave is found at a setting of 1515 m . Calculate the corresponding frequency setting. [speed of radio waves $=3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ ]
7. Sound travels at a speed of about $330 \mathrm{~m} \mathrm{~s}^{-1}$ in air. What is the wavelength range for sound waves audible to the human ear? [the audible frequency range is $20-20000 \mathrm{~Hz}$ ]
8. You can hear sound under water. Sound travels in water at a speed of $1500 \mathrm{~m} \mathrm{~s}^{-1}$. What is the wavelength range corresponding to the same frequency range as in question 7 ?

Waves - General

1. Explain the difference between a) longitudinal and transverse waves, b) mechanical and electromagnetic waves.
2. Into which of the above four categories would you put a) sound waves, b) light waves, c) water waves, d) radio waves?
3. Define the terms a) wavelength, b) frequency, c) amplitude, d) period as applied to waves.
4. State four different properties of light and sound waves.
5. Which of the following is increased when light waves pass from a dense to a less dense medium a) velocity, b) frequency, c) wavelength, d) amplitude?
6. What is the separation of compressions and rarefactions in a progressive sound wave?
particle
dispiacemeat $/ 1 \mathrm{~mm}$

7. a) The graph shows how particle displacement varies with distance for a sound wave at one instant in time. From the graph, determine:
i) The wavelength of the wave
ii) The amplitude of the wave
iii) The phase difference between the vibration of the particle at $A$ and that of the particle at $B$.
b) Explain what is meant by compression and rarefaction when applied to a sound wave travelling in air.
c) At this instant a compression occurs at B.
i) Use the graph to explain how such a compression arises.
ii) Copy the graph and indicate on your copy the positions of another compression and rarefaction.

## Refraction Question

3 Figure 2 shows three wavefronts of light directed towards a glass block in the air. The direction of travel of these wavefronts is also shown.

Complete the diagram to show the position of these three wavefronts after partial reflection and refraction at the surface of the glass block.



Figure 2

## Problems on Refracting

1. Which of the following statements is correct:

When monochromatic light travels from glass to air the refracted light compared to the incident light shows:
A. a decreased angle between its direction of propagation and the normal
B. a decreased speed
C. an increased frequency
D. an increased wavelength
E. an increased intensity
2. Plane parallel water waves are travelling at a speed of $4 \mathrm{~cm} \mathrm{~s}^{-1}$ in a ripple tank. They have a frequency of 30 Hz . They are incident at the boundary of a shallower area of water so that the incident wavefronts make an angle of $45^{\circ}$ with the boundary. If the speed in the shallower water is $3 \mathrm{~cm} \mathrm{~s}^{-1}$ calculate: (i) the angle made by the refracted wavefronts with the boundary; (ii) the wavelength of the incident waves; (iii) the wavelength of the refracted waves

Finally, complete the diagram below of the refracting waves.

3. To someone standing on the side, a swimming pool appears shallower than it really is. With the aid of a ray diagram explain why this should be so?

4. A ray of light is incident at an angle of $40^{\circ}$ on a rectangular glass block. If the block has a thickness of 0.2 m calculate the lateral (sideways) displacement of the ray emerging from the block. [refractive index of glass $=1.5$ ]
5. A ray of light has an angle of incidence of $20^{\circ}$ when travelling from water into glass. Given that the absolute refractive index of water is 1.33 and the absolute refractive index of glass is 1.50 , calculate:
(i) the angle of refraction in the glass
(ii) the ratio of the speed of the light in water to that in glass
(iii) the ratio of the wavelength of the light in water to that in glass

## Basic Exam Questions

The diagram below shows a liquid droplet placed on a cube of glass. A ray of light from air, incident normally on to the droplet, continues in a straight line and is refracted at the liquid to glass boundary as shown.
refractive index of the glass $=1.45$

(a) Calculate the speed of light
(i) in the glass,
$\qquad$
(ii) in the liquid droplet.
$\qquad$
(b) Calculate the refractive index of the liquid.
(c) On the diagram opposite, complete the path of the ray showing it emerge from the glass cubr into the air.
No further calculations are required.

Two prisms made from different glass are placed in perfect contact to form a rectangular block surrounded by air as shown.
Medium 1 has a smaller refractive index than medium 2.

(a) A ray of light in air is incident normally on medium 1 as shown. At the boundary between medium 1 and medium 2 some light is transmitted and the remainder reflected.
(i) Sketch, without calculation, the path followed by the refracted ray as it enters medium 2 and then emerges into the air.
(ii) Sketch, without calculation, the path followed by the reflected ray showing it emerging from medium 1 into the air.
(b) The refractive index of medium 1 is 1.40 and that of medium 2 is 1.60 .
(1) Give the angle of incidence at the boundary between medium 1 and medium 2.
(ii) Calculate the angle of refraction at this boundary.
(c) Calculate the critical angle for a ray passing from medium 2 into the air

The diagram shows a ray of light passing from air into a glass prism at an angle of incidence $\theta_{1}$. The light emerges from face BC as shown.
refractive index of the glass $=1.55$

(a) (i) Mark the critical angle along the path of the ray with the symbol $\theta_{\mathrm{s}}$.
(ii) Calculate the critical angle, $\theta_{\mathrm{c}}$.
(b) For the ray shown calculate the angle of incidence, $\theta_{1}$.
(c) Without further calculations draw the path of another ray of light incident at the same point on the prism but with a smaller angle of incidence.
The path should show the ray emerging from the prism into the air.

A glass plate surrounded by air is made up of two parallel sided sheets of glass in perfect contact as shown in the figure. Medium 1, the top sheet of glass, has a smaller refractive index than medium 2.

(a) A ray of light in air is incident on the top sheet of glass and is refracted at an angle of $40^{\circ}$ as shown in the figure. At the boundary between medium 1 and medium 2 some light is transmitted and the remainder reflected.

On the figure, sketch without calculation, the following:
(i) the path followed by the transmitted ray showing it entering from the air at the top and emerging into the air at the bottom;
(ii) the path followed by the reflected ray showing it emerging from medium 1 into the air.
(b) The refractive index of medium 1 is 1.35 and that of medium 2 is 1.65 .
(i) Calculate the angle of incidence where the ray enters medium 1 from the air.
(ii) Calculate the angle of refraction at the boundary between medium 1 and medium 2.
(c) Total internal reflection will not occur for any ray incident in medium 1 at the boundary with medium 2.

Explain, without calculation, why this statement is true.
$\qquad$
$\qquad$
$\qquad$

## Problems on Total Internal Reflection and Fibre Optics

1. Which of the following statements is correct?

When light travels in an optically more dense medium towards an optically less dense medium it:
A. is always refracted towards the normal
B. is always refracted away from the normal
C. travels at a lower speed on reaching the less dense medium
D. is partially internally reflected if the angle of incidence is less than the critical angle
E. is totally internally reflected if the angle of incidence is less than the critical angle
2. (a) Calculate the critical angle for a ray of light travelling from the bottom of a pond into the air above it
(b) If the surface of the pond now freezes calculate the new critical angle at the water/ice interface and the new critical angle at the ice/air interface
(c) At what angle would a ray of light have to be incident on the water/ice interface to just escape from the top surface of the ice?
(Refractive index of water $=1.33$, refractive index of ice $=1.30$ )
3. An optics fibre is often clad with material of a lower refractive index to prevent light passing from one fibre to an adjacent one in a coherent bundle.
(a) What effect would no cladding have on an image transmitted down a coherent bundle of fibres?
(b) If the refractive index of the cladding is 1.460 and the refractive index of the fibre material is 1.462 calculate the critical angle for light passing through the fibre incident on the interface between the core and the cladding
4. A diver working at a depth of 5 m has a bright point source of light. What is the diameter of the circle of light on the surface of the water seen by his colleagues in a boat?
[refractive index of water $=1.33$ ]
5. A ray of light just undergoes total internal reflection at the second face of a glass prism. If the prism angle is $60^{\circ}$ and the refractive index of the glass is 1.52 .
(a) Calculate the angle of incidence of the light when it is incident on the first face of the prism
(b) Calculate the total deviation of the ray of light from its original direction

## Problems on Polarisation

1. Which one of the following examples of wave types cannot be polarised?
A. x-rays
B. radio waves
C. microwaves
D. light waves
E. ultrasonic waves
2. A polarised wave is produced by vibrating one end of a rope vertically up and down. If the rope goes through a slit how must the slit be orientated to:
(a) allow the wave to pass through
(b) allow no wave to pass through?

Explain your answer
3. A light wave is travelling towards you. It has a wavelength of 600 nm and a speed of $3.0 \times 10^{8}$ $\mathrm{m} \mathrm{s}^{-1}$. The electric field of this light wave is polarised in the vertical direction. Describe, with the aid of graphs, how the magnetic field in the light wave varies with time and position.
4. T.V. and radio ærials have to be aligned in certain directions and in particular planes to receive the best signals. What does this tell you about transmitted T.V. and radio signals?
5. How could you use an analyser to distinguish between:
(a) partially polarised light
(b) unpolarised light
(c) completely polarised light?
6. (a) Describe and explain what is observed in the situation right when the analyser is slowly rotated through $360^{\circ}$



(b) How can polarised light be used to study stresses in transparent plastic?
(c) What applications does this have in industry?
7. The emitter in a 3 cm electromagnetic wave kit emits polarised electromagnetic waves. When a metal grid is set up in front of the emitter, the following results are obtained.


Emitter

i)

Grid with vertical metal bars


Emitter

ii)

Grid with horizontal metal bars

Are the 3 cm waves polarised in the vertical plane or in the horizontal plane?

Q1. Read through the following passage and answer the questions that follow it.

## Measuring the speed of sound in air

After the wave nature of sound had been identified, many attempts were made to measure its speed in air. The earliest known attempt was made by the French scientist Gassendi in the 17th century. The procedure involved timing the interval between seeing the flash of a gun and hearing the bang from some distance away. Gassendi assumed that, compared with the speed of sound, the speed of light is infinite. The value he obtained for the speed of sound was $480 \mathrm{~m} \mathrm{~s}^{-1}$. He also realised that the speed of sound does not depend on frequency.
A much better value of $350 \mathrm{~m} \mathrm{~s}^{-1}$ was obtained by the Italian physicists Borelli and Viviani using the same procedure. In 1740 another Italian, Bianconi, showed that sound travels faster when the temperature of the air is greater.
In 1738 a value of $332 \mathrm{~m} \mathrm{~s}^{-1}$ was obtained by scientists in Paris. This is remarkably close to the currently accepted value considering the measuring equipment available to the scientists at that time. Since 1986 the accepted value has been $331.29 \mathrm{~m} \mathrm{~s}^{-1}$ at $0^{\circ} \mathrm{C}$.
(a) Suggest an experiment that will demonstrate the wave nature of sound (line 1).
$\qquad$
$\qquad$
$\qquad$
(b) Using Gassendi's value for the speed of sound (line 6), calculate the time between seeing the flash of a gun and hearing its bang over a distance of 2.5 km .
$\qquad$time $=$ S
(c) Explain why it was necessary to assume that 'compared with the speed of sound, the speed of light is infinite' (line 5).
$\qquad$
$\qquad$
$\qquad$
(d) Explain one observation that could have led Gassendi to conclude that 'the speed of sound does not depend on frequency' (line 7).
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(e) Explain how the value obtained by Borelli and Viviani was 'much better' than that obtained by Gassendi (line 8).
$\qquad$
$\qquad$
(f) The speed of sound $c$ in dry air is given by

$$
c=k \sqrt{\theta+273.15}
$$

where $\theta$ is the temperature in ${ }^{\circ} \mathrm{C}$, and $k$ is a constant.
Calculate a value for $k$ using data from the passage.

$$
k=\ldots . \ldots . . . . . . . . . . . . . . . . . . \mathrm{m} \mathrm{~s}^{-1} K^{-1 / 2}
$$

(g) State the steps taken by the scientific community for the value of a quantity to be 'accepted' (line 13).
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Q1. a) Describe the structure of a step-index optical fibre outlining the purpose of the core and the cladding.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) A signal is to be transmitted along an optical fibre of length 1200 m . The signal consists of a square pulse of white light and this is transmitted along the centre of a fibre. The maximum and minimum wavelengths of the light are shown in the table below.

| Colour | Refractive index of fibre | Wavelength / nm |
| :---: | :---: | :---: |
| Blue | 1.467 | 425 |
| Red | 1.459 | 660 |

Explain how the difference in refractive index results in a change in the pulse of white light by the time it leaves the fibre.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Discuss two changes that could be made to reduce the effect described in part (b).
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
13. Waves on the open sea, known as gravity waves in order to distinguish them from ripples on a pond, have a speed $v$ that depends upon the wavelength $\lambda$ and the depth of the sea, $h$.

In deep water, $h \gg \lambda$ and the speed $v$ is independent of $h$, but does depend upon $\lambda$.

$$
v=\sqrt{\frac{g \lambda}{2 \pi}}
$$

In shallow water, $h \ll \lambda$, and the speed $v$ is independent of $\lambda$, but does depend upon $h$.

$$
v=\sqrt{g h}
$$

a) For a ship in deep water, the motion of the ship creates a wave such that the faster the speed the longer the wavelength. At some speed, known as the hull speed, $v_{\text {hull }}$, the wavelength becomes equal to the length of the ship $L$, as shown below. It is then very difficult for the ship to increase its speed as it has to climb the wave at the bow.


Figure 1

Show that $v_{\text {hull }}=1.2 L^{1 / 2}$
$\qquad$
b) The formula $v_{\text {hull }}=1.2 L^{1 / 2}$ only works when $L$ is measured in metres. Explain why.
$\qquad$
$\qquad$
c) Show that for deep water waves, $v=\frac{g}{2 \pi} T$ where $T$ is the period of the wave.
$\qquad$
$\qquad$
d) A Tsunami (a wave produced as the result of an earthquake) on the ocean has an immense wavelength of 80 km (so the shallow water situation applies). Calculate the speed of the wave when the depth of the ocean is 4.7 km , and also when it enters the coastal shallows where the depth is 10 m .
$\qquad$
$\qquad$
$\qquad$
$\qquad$
e) The power $P$ associated with a Tsunami wave progressing across the ocean is proportional to the speed of the wave, $v$ (the speed of energy flow), and the square of the amplitude $A$. The power flowing past a point is constant (otherwise energy would accumulate). Show that for the Tsunami, $A$ is proportional to $h^{-1 / 4}$.
$\qquad$
$\qquad$
$\qquad$
f) If the amplitude of the wave is 35 cm on the open ocean where the depth is 4.7 km , calculate the amplitude of the wave when the depth of the water is 10 metres.
$\qquad$
g) If the distant from the source of the Tsunami is only a few thousand kilometres then the Earth can be considered as a flat surface. However, if the distance from the source is very great then the curvature of the surface of the Earth will focus the waves. The intensity of the wave varies as $\frac{1}{\sin \left(\frac{r}{R}\right)}$ where $r$ is the distance from the source and $R$ is the radius of the Earth. At what distance from the source will the wave intensity begin to increase due to focusing?

$$
R=6,400 \mathrm{~km}
$$

Note that in $\sin (r / R)$ the term $r / R$ will give the angle in radians.
$\qquad$

## Waves Answers

Waves: Frequency, Time Period, Wavelength and Speed. Page 2.

1. a) 0.5 s
b) $33 \mu \mathrm{~s}$
c) 40 ns
d) 0.25 ns
2. a) $7.1 \times 10^{2} \mathrm{~Hz}$
b) $8.3 \times 10^{-3} \mathrm{~Hz}$
c) $2.1 \times 10^{7} \mathrm{~Hz}$
d) $2.0 \times 10^{5} \mathrm{~Hz}$
3. a) 7.3 ms
b) $138(140) \mathrm{Hz}$
4. 

| Frequency | Velocity | Wavelength |
| :--- | :--- | :--- |
| 15 Hz | $120 \mathrm{~ms}^{-1}$. | 8.0 m |
| 3.8 MHz | $5000 \mathrm{~ms}^{-1}$. | 1.3 mm |
| $1.2 \times 10^{15} \mathrm{~Hz}$ | $2.0 \times 10^{8} \mathrm{~ms}^{-1}$. | 0.17 mm |
| 230 Hz | $340 \mathrm{~ms}^{-1}$ | 1.48 m |
| 11 GHz | $3.0 \times 10^{8} \mathrm{~ms}^{-1}$. | 2.8 cm |

5. Frequency - number of oscillations per second (Hz).

Time Period - the time taken for one complete oscillation (s).
Wavelength - the distance between the same point on two consecutive oscillations (m).
Wave speed - the speed at which the wave travels through the medium $\left(\mathrm{ms}^{-1}\right)$.

Problems on Describing Waves. Page 3.

1. i) 10 mm
ii) 4 mm
2. i) 10 mm
ii) 0.5 s
iii) 2 Hz
3. $8 \mathrm{mms}^{-1}$.
4. $6 \times 10^{14} \mathrm{~Hz}$.
5. ...
6. $\quad 198 \mathrm{kHz}$.
7. $0.0165 \mathrm{~m} \rightarrow 16.5 \mathrm{~m}$.
8. $0.075 \mathrm{~m} \rightarrow 75 \mathrm{~m}$.

Waves - General. Page 4.
1-6 (Look in notes).
7. a) i) 1.33 m
ii) $5 \mu \mathrm{~m}$
iv) $135^{\circ}$

Problems on Refracting. Page 7.

1. D
2. i) $58^{\circ}$
ii) $0.13 \mathrm{~cm}, 0.1 \mathrm{~cm}$
3. ...
4. 0.086 m
5. i) $17.7^{\circ}$
ii) 1.13
iii)1.13

Basic Exam Questions. Page 8.

1. a) i) $2.07 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
ii) $2.25 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$
b) 1.33
c) ray refracts away from the normal at the glass-air boundary
2. a) i) Normal drawn on med1-med2 boundary. Ray refracts towards the normal.
ii) ray reflects at med1-med2 boundary, angle reflection = angle incidence. At med1-air boundary ray refracts away from the normal.
b) i) $20^{\circ}$
ii) $17.4^{\circ}$
c) $38.7^{\circ}$
3. a) i) critical angle $=$ angle of incidence at BC boundary
ii) $40.2^{\circ}$
b) $14.8^{\circ}$
c) at AB boundary refracts towards normal, hits BC boundary closer to C than first ray. At BC boundary angle incidence is greater than critical angle therefore TIR, angle reflection = angle incidence. At AC boundary angle incidence less than cri tang so refraction away from the normal.
4. a) i)med1-med2 boundary refraction towards normal. At med2-air boundary refraction away from normal.
ii) reflection angle $=$ angle incidence. At med1-air boundary refraction away from normal.
b) i) $60.2^{\circ}$
ii) $31.7^{\circ}$
c) TIR only occurs at a boundary with a less optically dense medium.

Problems on TIR and Fibre Optics. Page 12.

1. D.
2. a) $48.8^{\circ}$, b) $77.8^{\circ}, 50.3^{\circ}$, c) $48.8^{\circ}$.
3. $87.0^{\circ}$.
4. 11.4 m
5. a) $29.4^{\circ}$, b) $119 \circ$

Problems on Polarisation. Page 13.

1. E
2. a) vertically
b) horizontally.

## Comprehension Question - Answers.

Q1. (a) Suitable experiment eg diffraction through a door / out of a pipe $\checkmark$
(b) Using $\mathrm{c}=\mathrm{d} / \mathrm{t}$ $t=2500 / 480=5.2 \mathrm{~s} \checkmark$
(c) (Measured time is difference between time taken by light and time taken by sound)

Calculation assumes that light takes no time to reach observer, ie speed is infinite $\checkmark$ Do not allow "could not know speed of light"
(d) Sound from gun is a mixture of frequencies.

Alternative for $1^{\text {st }}$ mark '(so speed is independent of frequency) the sound of the gun is similar when close and far away'

All the sound reaches observer at the same time,
(e) More accurate, as it is closer to the accepted value.
(f) When $\theta=0{ }^{\circ} \mathrm{C} \quad \mathrm{c}=331.29 \mathrm{~m} \mathrm{~s}^{-1}$

Therefore
$331.29=k \sqrt{ } 273.15 \checkmark$
$\mathrm{k}=20.045 \checkmark$
(g) The method and value are published $\checkmark$
other scientists repeat the experiment using the same method

## Extended Writing Question - Answers.

Q1. a) Core is transmission medium for em waves to progress (by total internal reflection) $\checkmark$ Allow credit for points scored on a clear labelled diagram.

Cladding provides lower refractive index so that total internal reflection takes place

And offers protection of boundary from scratching which could lead to light leaving the core.
(b) Blue travels slower than red due to the greater refractive index

Red reaches end before blue, leading to material pulse broadening
The first mark is for discussion of refractive index or for calculation of time difference.

Alternative calculations for first mark
Time for blue $=d / v=d /(c / n)=1200 /\left(3 \times 10^{8} / 1.467\right)=5.87 \times 10^{6} s$
Time for red $=d / v=d /(c / n)=1200 /\left(3 \times 10^{8} / 1.459\right)=5.84 \times 10^{-6} s$
Time difference $=5.87 \times 10^{-6}-5.84 \times 10^{-6}=3(.2) \times 10^{-8} \mathrm{~s}$
The second mark is for the link to material pulse broadening
(c) Discussions to include:

Use of monochromatic source so speed of pulse constant
Use of shorter repeaters so that the pulse is reformed before significant pulse broadening has taken place.

Use of monomode fibre to reduce multipath dispersion
Answer must make clear that candidate understands the distinction between modal and material broadening.
13. Waves on the open sea, known as gravity waves in order to distinguish them from ripples on a pond, have a speed $v$ that depends upon the wavelength $\lambda$ and the depth of the sea, $h$.

In deep water, $h \gg \lambda$ and the speed $v$ is independent of $h$, but does depend upon $\lambda$.

$$
v=\sqrt{\frac{g \lambda}{2 \pi}}
$$

In shallow water, $h \ll \lambda$, and the speed $v$ is independent of $\lambda$, but does depend upon $h$.

$$
v=\sqrt{g h}
$$

a) For a ship in deep water, the motion of the ship creates a wave such that the faster the speed the longer the wavelength. At some speed, known as the hull speed, $v_{\text {hull }}$, the wavelength becomes equal to the length of the ship $L$, as shown below. It is then very difficult for the ship to increase its speed as it has to climb the wave at the bow.


Figure 1

Show that $v_{\text {hull }}=1.2 L^{1 / 2}$
$\sum^{v}=\sqrt{\frac{g \lambda}{2 \pi}}$ and with $\lambda=L$ $\qquad$
$-\sqrt{\frac{g}{2 \pi}}=1.25=1.2$ $\qquad$
b) The formula $v_{\text {hull }}=1.2 L^{1 / 2}$ only works when $L$ is measured in metres. Explain why.
$\qquad$ we have substituted a specific unit dependent value for $g$ $\qquad$ $\checkmark$ $\qquad$
$\qquad$ which is in metres $\qquad$ $\checkmark$ $\qquad$ [2]
c) Show that for deep water waves, $v=\frac{g}{2 \pi} T$ where $T$ is the period of the wave.
$L^{2}=\sqrt{\frac{g \lambda}{2 \pi}}$ so $^{2}=\frac{g}{2 \pi} \frac{v}{f}$ and hence $v=\frac{g T}{2 \pi}$ $\qquad$ $\checkmark$ $\qquad$
d) A Tsunami (a wave produced as the result of an earthquake) on the ocean has an immense wavelength of 80 km (so the shallow water situation applies). Calculate the speed of the wave when the depth of the ocean is 4.7 km , and also when it enters the coastal shallows where the depth is 10 m .
$\qquad$ $v=\sqrt{g h}$
$\qquad$ for $h=4.7 \mathrm{~km}$ then $v=215 \mathrm{~m} \mathrm{~s}^{-1}=2.2 \times 10^{2} \mathrm{~m} \mathrm{~s}^{-1}$ $\qquad$
$\qquad$
$\qquad$ for $h=10 \mathrm{~m}$ then $v=9.9 \mathrm{~m} \mathrm{~s}^{-1}$ $\qquad$ $\checkmark$ $\qquad$
e) The power $P$ associated with a Tsunami wave progressing across the ocean is proportional to the speed of the wave, $v$ (the speed of energy flow), and the square of the amplitude $A$. The power flowing past a point is constant (otherwise energy would accumulate). Show that for the Tsunami, $A$ is proportional to $h^{14}$.
$\qquad$ $P \alpha v A^{2}$ and if the power is constant then $v \alpha l / A^{2}$ $\qquad$ $\checkmark$ $\qquad$
$\qquad$ But $v=\sqrt{g h}$ so $\sqrt{h} \alpha A^{-2}$ and hence $h^{\frac{1}{4}} \alpha \frac{1}{A}$ $\qquad$ $\checkmark$ $A$ proportional to $h$ to power (-1/4)
f) If the amplitude of the wave is 35 cm on the open ocean where the depth is 4.7 km , calculate the amplitude of the wave when the depth of the water is 10 metres.

$$
A=k h^{-\frac{1}{4}} \quad \text { So } \frac{A}{0.35}=\left(\frac{10}{4700}\right)^{-\frac{1}{4}} \quad A=1.6 \mathrm{~m} \quad \checkmark
$$

$$
\begin{equation*}
\text { (or } \left.A_{1}^{4} h_{1}=A_{2}^{4} h_{2} \quad \text { so }(0.35)^{4} \times 4700=A_{2}^{4} \times 10 \quad\right) \tag{1}
\end{equation*}
$$

g) If the distant from the source of the Tsunami is only a few thousand kilometres then the Earth can be considered as a flat surface. However, if the distance from the source is very great then the curvature of the surface of the Earth will focus the waves. The intensity of the wave varies as $\frac{1}{\sin \left(\frac{r}{R}\right)}$ where $r$ is the distance from the source and $R$ is the radius of the Earth. At what distance from the source will the wave intensity begin to increase due to focusing?

$$
R=6,400 \mathrm{~km}
$$

Note that in $\sin (r / R)$ the term $r / R$ will give the angle in radians.

When the factor $\sin (r / R)=1$ then the intensity factor changes from the intensity decreasing to the intensity increasing. $\qquad$
This is when $\pi / 2=r / R$ $\qquad$ $\checkmark$ $\qquad$
At $r=R \pi / 2$ $\qquad$ $R=10,053 \mathrm{~km}=1.0 \times 10^{4} \mathrm{~km}$ $\qquad$ $\checkmark$ $\qquad$ [2]

